NONSTATIONARY SHOCK WAVES IN A LOW-DENSITY PLASMA

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Numerical methods are used to study nonstationary collisionless shocks in a plasma propagating at some arbitrary angle to the unperturbed magnetic field in cases where the electrical conductivity and electron thermal conductivity have finite values.

Effects related to collisionless shock waves in a low-density plasma have been studied experimentally and theoretically; numerical methods of modeling on a computer have been used in [1-7]. The structure of the shock waves is determined by the nonlinearity, dispersion, and dissipation effects. Critical values of the Mach number M_* have been determined at which there is a qualitative change in this structure.

Morton [4] has studied in detail both stationary and (to a lesser extent) nonstationary compression waves in a two-fluid plasma with an arbitrarily oriented magnetic field; he does not include dissipation of the energy required for the formation of shock waves or thermal conductivity. In [6, 7], an analysis of the singular points of the equations describing the structure of the shock waves leads to the critical parameters for which the solution becomes discontinuous. The solutions of the structure equations are given in [7]. The propagation of nonstationary shock waves across a magnetic field is considered in [5] where the effect of electrical conductivity and electron thermal conductivity are taken into account and the isomagnetic jump in density for an almost constant magnetic field is studied.

This paper is devoted to a study by numerical methods in the two-fluid hydrodynamic approximation of the structure and critical parameters of nonstationary shock waves propagating in a low-density plasma at an arbitrary angle to the unperturbed magnetic field in cases where dispersion, electrical conductivity, and electron thermal conductivity are taken into account.

We take the wave propagating along the x axis and assume that the unperturbed magnetic field $H_0 = \{H_x, 0, H_z\}$ lies in the xz plane, making an angle θ with the z axis. The initial system of equations can be written

$$
\begin{gathered}
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \\
nm_i\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial}{\partial x}\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right) \\
nm_i\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x}\right) = \frac{H_x}{4\pi}\frac{\partial H_y}{\partial x} \\
nm_i\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x}\right) = \frac{H_x}{4\pi}\frac{\partial H_z}{\partial x} \\
\frac{\partial H_y}{\partial t} = -\frac{\partial}{\partial x}\left\{uH_y - vH_x + \frac{m_e c^2}{4\pi e^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(\frac{1}{n}\frac{\partial H_y}{\partial x}\right) + \frac{cH_x}{4\pi en}\frac{\partial H_z}{\partial x} + \frac{c^2}{4\pi\sigma}\frac{\partial H_y}{\partial x}\right\} \\
\frac{\partial H_z}{\partial t} = \frac{\partial}{\partial x}\left\{wH_x - uH_z + \frac{m_e c^2}{4\pi e^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(\frac{1}{n}\frac{\partial H_z}{\partial x}\right) + \frac{c^2}{4\pi\sigma}\frac{\partial H_z}{\partial x} - \frac{cH_x}{4\pi en}\frac{\partial H_y}{\partial x}\right\} \\
\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \gamma p\frac{\partial u}{\partial x} = (\gamma - 1)\left\{\frac{c^2}{16\pi^2\sigma}\left[\left(\frac{\partial H_y}{\partial x}\right)^2 + \left(\frac{\partial H_z}{\partial x}\right)^2\right] + \frac{\partial}{\partial x}\left(\varkappa_1\frac{\partial T}{\partial x}\right)\right\} \\
H_x = H_0 \sin\theta
\end{gathered} \tag{1}
$$

Here $\mathbf{u} = \{u, v, w\}$ is the macroscopic velocity of the plasma, $\sigma = ne^2/m_e\nu$ is the conductivity, $\varkappa_1$ is the electron thermal conductivity, γ is the adiabatic coefficient, ν is the effective collision frequency of

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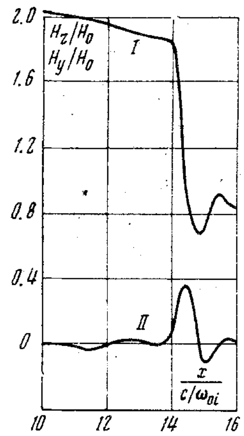

Fig. 1

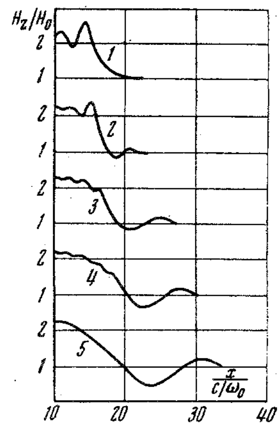

Fig. 2

the plasma particles with the fluctuations in the electromagnetic field (taken to be constant), $p = nT$ is the electron pressure (the ions are assumed cold).

In order to solve the nonstationary problem we write (1) in dimensionless variables and Lagrangian coordinates

$$
\begin{gathered}
\frac{\partial u}{\partial \tau} = -\frac{1}{2}\frac{\partial}{\partial \xi}(H_y^2 + H_z^2 + p), \qquad \frac{\partial v}{\partial \tau} = \sin\theta \frac{\partial H_y}{\partial \xi} \\
\frac{\partial w}{\partial \tau} = \sin\theta \frac{\partial H_z}{\partial \xi}, \qquad u = \frac{\partial x}{\partial \tau}, \qquad V = \frac{\partial x}{\partial \xi} \\
\frac{\partial}{\partial \tau}(VH_y) = \sin\theta \frac{\partial v}{\partial \xi} + \varkappa \frac{\partial^2 H_y}{\partial \xi^2} + \sin\theta \frac{\partial^2 H_z}{\partial \xi^2}\frac{1}{\sqrt{\beta}} + \frac{\partial^3 H_y}{\partial \tau\, \partial \xi^2} \\
\frac{\partial}{\partial \tau}(VH_z) = \sin\theta \frac{\partial w}{\partial \xi} + \varkappa \frac{\partial^2 H_z}{\partial \xi^2} - \sin\theta \frac{\partial^2 H_y}{\partial \xi^2}\frac{1}{\sqrt{\beta}} + \frac{\partial^3 H_z}{\partial \tau\, \partial \xi^2} \\
V\frac{\partial p}{\partial \tau} + p\gamma \frac{\partial V}{\partial \tau} = 2(\gamma - 1)\left\{\left(\frac{\partial H_z}{\partial \xi}\right)^2 + \left(\frac{\partial H_y}{\partial \xi}\right)^2 + \frac{1}{2}\chi \frac{\partial}{\partial \xi}\left(\frac{1}{V^2}\frac{\partial T}{\partial \xi}\right)\right\} \\
\left(\beta = \frac{m_e}{m_i}, \qquad \omega_* = \frac{eH_0}{\sqrt{m_e m_i}\, c}, \qquad \omega_0 = \sqrt{\frac{4\pi n e^2}{m_e}}, \right. \\
\left. V_A = \frac{H_0}{\sqrt{4\pi n_0 m_i}}, \qquad \varkappa = \frac{\nu}{\omega_*}, \qquad \chi = \frac{\varkappa_1}{n}\right)
\end{gathered} \tag{2}
$$

Here the magnetic field components $H_{y,\,z}$, the pressure p, the Eulerian and Lagrangian coordinates x, ξ, the time τ, the velocity components u, v, w, and the specific volume V are measured in units of H_0, $H_0^2/8\pi$, c/ω_0, ω_*^{-1}, V_A, and n_0, respectively, where ω_* is the hybrid frequency, V_A is the Alfvén velocity, and the value of χ is taken to be constant.

We assume that at the initial moment of time a uniform cold plasma with $p_0 \ll H_0^2/8\pi$ and density n_0 occupies the region $0 \le x \le x_{max}$ $(0 \le \xi \le \xi_{max})$ and that at the left boundary of this region the magnetic field increases with time in some definite way; we can write the initial and boundary conditions as

$$
\begin{gathered}
x(\xi, 0) = \xi, \quad V(\xi, 0) = 1, \quad H_z(\xi, 0) = \cos\theta \\
H_y(\xi, 0) = p(\xi, 0) = u(\xi, 0) = v(\xi, 0) = w(\xi, 0) = T(\xi, 0) = 0 \\
H_z(0, \tau) = 1 + A(1 - e^{-\omega\tau}), \\
p(0, \tau) = \frac{\partial T}{\partial \xi}(0, \tau) = 0
\end{gathered} \tag{3}
$$

$$
\frac{\partial H_y}{\partial \xi}(\xi_{max}, \tau) = \frac{\partial H_z}{\partial \xi}(\xi_{max}, \tau) = \frac{\partial (u, v, w)}{\partial \xi}(\xi_{max}, \tau) = 0 \tag{4}
$$

where ω is the frequency of the external field in units of ω_* and A is the amplitude of this field in units of H_0.

The finite-difference analog of the differential problem (2)-(4) was programmed on a BÉSM-6 computer.

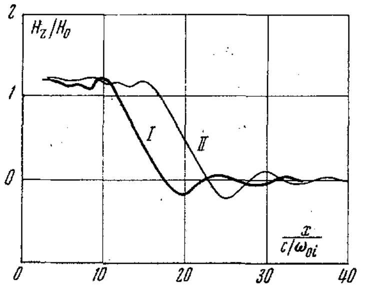

Fig. 3

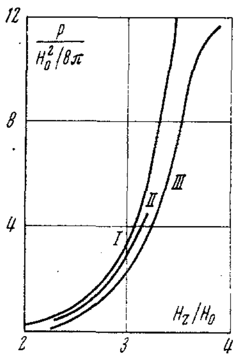

Fig. 4

We consider the results obtained for subcritical Mach numbers ($M < M_*$).

It is shown in [5] that a shock wave propagating across the magnetic field ($\theta=0$) is quasistationary for $M < 2.5$ and that allowance for thermal conductivity produces only a very small increase in the width of the front. In this paper we are mainly interested in waves traveling at some angle to the unperturbed field (oblique waves with $\theta \neq 0$, $\theta \neq 90°$) or along it (longitudinal waves with $\theta = 90°$).

Consider the case $\sqrt{\beta} \ll \theta < \pi/2$. It follows from the dispersion law for oblique waves (see, for example, [8]) that the shock wave profile has an oscillatory leading train (or precursor). Typical quasistationary profiles of the transverse magnetic field components (I is H_z and II is H_y) for an oblique shock wave are shown in Fig. 1. The spatial scale of the oscillations depends on the angle θ and its order of magnitude can be estimated from the equation $\delta \sim c\theta/\omega_{0i}$. The total width of the front including the train depends on the wave velocity, the angle θ, and the amount of dissipation and can be estimated from the equation $\Delta \sim V_A M\theta^2/\nu\beta$ [2]. Calculations for $A=2$ and $\varkappa=8$ give for $\theta=30$, 45, and 60° the values $\delta=(0.8, 1.2,$ and $1.8)c/\omega_{0i}$, and $\Delta=(3.5, 6,$ and $9.5)c/\omega_{0i}$, respectively (ω_{0i} is the ion plasma frequency). There is a phase difference between the magnetic field components and its value can be found from the equation $\tan\varphi \sim M\theta\cdot(1-\theta^2/M^2)\varkappa$. Thus $\varphi \approx 70°$ for the case $M=1.3$, $\theta=30°$, $\varkappa=8$. The magnetic field profile leads the density profile near the front by a distance

$$L \sim c^2 / 4\pi\sigma V_A (M - 1) < c\theta / \omega_{0i}$$

as a result of the resistive dissipation mechanism [2]. When $A=2$, $\theta=30°$, and $\varkappa=8$, for example, $L \simeq 0.2\ c/\omega_{0i}$. An increase in the effective collision frequency ν (or $\varkappa = \nu/\omega_*$) changes the shape of the profile from oscillatory to monotonic. The oscillations disappear when the dispersion size $c\theta/\omega_{0i}$ becomes comparable to the dissipative size $c^2/(4\pi\sigma V_A M)$. Thus when $A=2$, $\theta=30°$, the profile becomes monotonic for $\varkappa = 16$ ($\nu=16\omega_*$).

The shock wave structure changes as the angle θ gets smaller. When $\theta \gg \beta^{1/2}$ the dispersion is related to the anisotropy of the plasma (ion dispersion), and when $\theta \ll \beta^{1/2}$ it is caused by the electron inertia (electron dispersion). The dispersion laws are quite different for these two cases (ω/k increases with the wave number k in the first case but decreases in the second). The shock wave structure is therefore different in these limiting cases (oscillations leading or lagging). As the value of θ approaches $\theta = \beta^{1/2}$, the shock wave takes on an intermediate structure: oscillations with characteristic size $\delta \sim c/\omega_0$ behind the front and oscillations with $\delta \sim c/\omega_{0i}$ beyond the front.

Figure 2 shows the transformation in the magnetic field profile as the angle θ between the plane of the front and the direction of the unperturbed magnetic field is altered. The curves denoted by the numbers 1, 2, 3, 4, and 5 correspond to values of $\theta=0$, 2.5, 4, 5, and 6°. As the angle θ is reduced, there is a development of the oscillatory structure behind the front and a decrease in the leading oscillations. For $M \simeq 2$, a double front structure is observed for angles of θ lying in the range $\theta_{min} \leq \theta \leq \theta_{max}$, where $\theta_{min} \approx 2°$, $\theta_{max} \approx 6°$. An increase in the Mach number produces a rise in θ_{min}. Thus, for $M \approx 2.3$, $\theta_{min} \approx 2.5°$.

We consider now the propagation of shock waves along H_0 ($\theta=90°$) at comparatively low frequencies $\omega \sim \omega_i = eH_0/m_i c$ ("switch-on" wave). The amplitude and velocity of switch-on shock waves are bounded from above by the values $\approx 1.5H_0$ and $\simeq 2V_A$ [9]. A typical profile for the magnetic field of a shock wave propagating along H_0 is shown in Fig. 3. Curve I corresponds to the instant of time $t=13\omega_i^{-1}$ and curve II to $t=17\omega_i^{-1}$ ($M \simeq 1.2$, $\omega=0.5\ \omega_i$, $A=1.3$). The profile is actually the superposition of two waves because in the linear approximation two waves with frequencies $\omega \sim \omega_i$ can propagate along the magnetic field; these waves have right- and left-handed circular polarization. The dispersion laws for the two waves are different so that the magnetic field profile in a switch-on shock contains oscillations both behind the front and beyond it. The faster wave, which has a resonance at $\omega = eH_0/m_ec$, leads the slower wave (with resonance at $\omega = eH_0/m_ic$) and there is a spatial separation of the shock wave profile into two parts which differ in the nature of their oscillations. A decrease in the frequency of the external field (to values of the order of $0.1\ \omega_i$, for example) makes the two phase velocities approach each other so that the profile separation no longer occurs.

We now consider the case of large Mach numbers. An increase in the Mach number produces a change in the shock wave structure as a result of the increased effect of nonlinearity and nonstationarity. For transverse propagation including the effect of thermal conductivity there is a quasistationary isomagnetic jump in density for Mach numbers $2.8 \leqslant M \leqslant 3.3$, and for $M \geqslant 3.4$ the shock wave structure is broken up [5]. Calculations show that for oblique and longitudinal waves at sufficiently high Mach numbers there is a continuous increase (in the absence of thermal conductivity) in the curvature of the density profile and the x-component of the particle velocity. The solution approaches discontinuity in these functions. The values of the critical Mach number M_* vary with the angle θ as shown below:

$\theta°$	0	30	45	60	85	90
M_*	2.8	2.6	2.3	2.1	1.65	1.6

Thus, as the angle θ between the plane of the front and the direction of the unperturbed magnetic field H_0 increases, there is a decrease in the value of the critical Mach number M_* at which break-up of the shock waves is observed.

Allowance for the electron thermal conductivity leads, for oblique waves, to the appearance of an isomagnetic jump whose width is determined by the coefficient of thermal conductivity. The critical Mach numbers $M*$ for fixed θ are higher than those obtained when thermal conductivity can be neglected. Calculations for $\chi = 0.1\varkappa$ for angles $\theta = 0$ and 30° give $M_* = 3.5$ and 2.9, respectively.

An analysis of the results for the case where $\theta = 90°$ shows that an increase of the Mach number of a switch-on shock leads to a continuous decrease in the amplitude of the magnetic field and a sharp rise in the gasdynamic pressure. The critical parameters of a switch-on shock at which inversion occurs have been obtained when the amplitude of the external field at the plasma boundary $A = 1.7$ and are equal to $M_* \simeq 1.6$ and $H_* \simeq 1.0$.

Let us consider the effect of electron heating behind the wave front. Experimental results [3, 10] and theoretical predictions [1] indicate the preferential heating of the electron plasma component in the front by a collisionless shock wave, and so in the case of (2)-(4) we take into account only the electron pressure, i.e., we assume that $T_e \gg T_i$ throughout.

Figure 4 shows the solution of the nonstationary problem (2)-(4) for $\theta = 30°$ (curve II), the experimental variation of pressure behind the shock front with the amplitude of the magnetic field (obtained on the UN-4 apparatus at the Institute of Nuclear Physics, Siberian Branch, Academy of Sciences of the USSR [10], curve III) and the Hugoniot adiabatic (curve I) which characterizes the variation of the total gasdynamic pressure $p = p_e + p_i$ with the amplitude of the shock. It can be seen that the electron pressure $p_e = nT_e$ is almost equal to the total pressure of the plasma. This confirms the theoretical prediction that there is preferential heating of the electron component when $M < M_*$ and $H < H_*$. Similar conclusions can be drawn for shock waves propagating across the unperturbed magnetic field [3].

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